It is again convenient to use the differential form of (5), which after elimination of T_s^*, becomes equation (18). The initial condition on q^* is obtained from the earlier solution, while initially, $T_b^* = 1$.

Schneider [3] has shown that for $Q^* = 0$, the equations for a finite slab have an analytical solution. However, for the case of constant $Q^*(\neq 0)$ he considered only the semi-infinite case. As an example of a finite slab we have considered his heat input ($Q^{*1/4} = 0{\cdot}05$) with an arbitrary but computationally convenient value of $\mathscr{L} = 3{\cdot}015$. Of course the method of solution, which was again numerical integration, is valid for non-constant Q^* also. The resulting solutions for T_b^* and T_s^* are shown in Fig. 1. The initial discontinuity in the slopes which occurs at $\tau \approx 1{\cdot}15$ (which is when $s = \mathscr{L}$) is a consequence of approximations inherent in the integral method.

CLOSURE

We have considered herein only problems involving a radiation boundary condition at the surface. However, it should be evident that similar starting solutions may be derived for fairly general forms of $q^*(T_s^*, \tau)$ provided that q^* is analytic in its arguments near $\tau = 0$.

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THE ECKERT REFERENCE FORMULATION APPLIED TO HIGH-SPEED LAMINAR BOUNDARY LAYERS OF NITROGEN AND CARBON DIOXIDE

H. A. SIMON, C. S. LIU and J. P. HARTNETT
University of Illinois at Chicago Circle, Chicago, Illinois

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NOMENCLATURE

C_f,	$\tau_w/\frac{1}{2}\rho\, u_s^2$ = local skin friction coefficient;
C_p,	specific heat at constant pressure;
h,	$q_w/(T_r - T_w)$ = local heat-transfer coefficient;
h_i,	$q_w/(i_r - i_w)$ = local enthalpy difference heat-transfer coefficient;
i,	enthalpy;
Pr,	Prandtl number;
q,	heat flow per unit time and area;
r,	$(T_r - T_s)/(u_s^2/2C_p)$ = recovery factor;
r_i,	$(i_r - i_s)/(u_s^2/2)$ = enthalpy recovery factor;
St_i,	$h_i/\rho u_s$ = Stanton number based on enthalpy difference;
T,	temperature;
u,	velocity.

Greek symbols

μ,	viscosity;
ρ,	density;
τ,	shearing stress.

Subscripts

i,	based on enthalpy;
r,	recovery;
s,	in free stream;
w,	at wall.

Superscripts

*	reference condition.

DESIGN calculations aimed at the determination of heat transfer, skin friction or recovery temperature in high-speed laminar boundary layers, can be extremely complex due to property variations. Considerable simplification is achieved if a constant property analysis can be used with the properties determined at suitably defined reference conditions.

In reference [1], Eckert presents the following expression for the reference temperature applicable to laminar boundary layers on surfaces at constant temperature and pressure, and for a wide range of Mach numbers:

$$T^* = T_s + 0{\cdot}50(T_w - T_s) + 0{\cdot}22(T_r - T_s). \qquad (1)$$

The constants in this equation were determined such that the reference temperature so defined, and used in a constant property analysis, gave good agreement with all the exact calculations available at that time. These calculations were concerned with air and were well approximated by the above approach regardless of the specific property variations assumed. This suggests that the relationship assumed for the reference temperature could also work for gases other than air, and the present work confirms this for N_2 and CO_2.

Laminar boundary-layer calculations have been carried out for N_2 and CO_2 streams allowing for variable properties [2] thus providing the opportunity for checking the assumed reference expression on gases other than air. The property calculations are described in reference [3].

Reference [1] gives tables in which the constant property results are compared with the exact calculations of Young and Janssen [4]. Percentage errors are also given. To simplify comparisons the same format has been followed here for N_2 and CO_2, using the same parameters.

The tables are arranged as follows:

Table 1. Wall at recovery temperature
(a) Nitrogen
(b) Carbon dioxide

Table 2. Constant wall temperature
(a) Nitrogen
(b) Carbon dioxide

Table 1, column 3 and Table 2, column 4 give values of the skin friction coefficient, $[C_f\sqrt{(Re)}]_s$ as obtained from the exact solutions. The following column in each table reports values of this same parameter $[C_f\sqrt{(Re)}]_s$ obtained however by application of the reference temperature method (for this reason this column is labelled $[C_f\sqrt{(Re)}]_s^*$). To obtain these values the following formulation has been used:

$$[C_f\sqrt{(Re)}]_s^* = [C_f\sqrt{(Re)}]^* \sqrt{\left(\frac{\rho^*\mu^*}{\rho_s\mu_s}\right)} = 0{\cdot}664 \sqrt{\left(\frac{\rho^*\mu^*}{\rho_s\mu_s}\right)}. \qquad (2)$$

Here we have used one of the basic reference temperature relationships, which is derived from a constant property analysis:

$$[C_f\sqrt{(Re)}]^* = 0{\cdot}664. \qquad (2a)$$

The difference between the two tabulated values of $[C_f\sqrt{(Re)}]_s$ though in a few instances larger than in reference 1, is still relatively small, being less than 2 or 3 per cent for both cases.

The recovery temperature is calculated from

$$T_r^* = T_s + \sqrt{(Pr^*)}\,\frac{u_s^2}{2C_p^*} \qquad (3)$$

where the Prandtl number and the specific heat are evaluated at the reference temperature defined by equation (1), it is compared in columns 6 and 7 of Table 1 with the value of the recovery temperature T_r found from the boundary-layer analysis. The percentage errors are no larger than those in reference 1.

In applying the reference temperature method to the

Table 1. *Recovery case*

T_s(°K)	M_s	$[C_f\sqrt{(Re)}]_s$	$[C_f\sqrt{(Re)}]_s^*$	per cent	T_r(°K)	T_r^*(°K)	per cent	r_i	$\sqrt{(Pr^*)}$	per cent
					(a) N_2					
218	4	0·574	0·569	−0·9	791	790	−0.1	0·836	0·846	1·2
555	4	0·564	0·560	−0·7	1843	1812	−1·7	0·839	0·852	1·5
1110	4	0·574	0·568	−1·0	3409	3382	−0·8	0·841	0·851	1·2
218	8	0·485	0·481	−0·8	2274	2199	−3·3	0·835	0·853	2·2
218	12	0·432	0·426	−1·4	4570	4438	−2·9	0·835	0·852	2·0
					(b) CO_2					
218	4	0·640	0·640	0	634	610	−3·8	0·854	0·860	0·7
555	4	0·612	0·610	−0·3	1271	1248	−1·8	0·834	0·842	1·0
1110	4	0·607	0·604	−0·5	2329	2317	−0·5	0·827	0·836	1·1
218	8	0·575	0·578	0·5	1554	1452	−6·6	0·832	0·841	1·1
555	8	0·536	0·534	−0·4	3103	3020	−2·7	0·821	0·834	1·6
218	12	0·521	0·522	0·2	2917	2752	−5·7	0·821	0·836	1·8

Table 2. Constant wall temperature

T_s(°K)	T_w(°K)	M_s	$[C_f\sqrt{(Re)}]_s$	$[C_f\sqrt{(Re)}]_s^*$	per cent	$(C_f/2St)^*$	$(Pr^*)^{\frac{2}{3}}$	per cent	$(C_f/2St_i)$	per cent
(a) N_2										
218	436	0	0·633	0·612	−3·3	0·806	0·804	−0·2	0·807	0·1
218	1308	0	0·553	0·547	−1·0	0·798	0·801	0·4	0·801	0·4
1110	555	0	0·707	0·693	−2·0	0·811	0·801	−1·2	0·810	−0·1
218	436	4	0·607	0·597	−1·6	0·778	0·800	2·8	0·800	2·8
218	1308	4	0·540	0·529	−2·0	0·776	0·799	3·0	0·799	3·0
555	555	4	0·624	0·615	−1·4	0·768	0·800	4·2	0·806	4·9
555	1110	4	0·592	0·582	−1·7	0·776	0·802	3·4	0·807	4·0
555	1665	4	0·571	0·559	−2·1	0·782	0·806	3·1	0·805	2·9
1110	555	4	0·651	0·638	−2·0	0·791	0·806	1·9	0·809	2·3
218	436	8	0·555	0·542	−2·3	0·749	0·801	6·9	0·802	7·1
218	1308	8	0·513	0·503	−1·9	0·764	0·803	5·1	0·802	5·0
218	436	12	0·509	0·493	−3·1	0·771	0·805	4·4	0·803	4.2
218	1308	12	0·485	0·473	−2·5	0·776	0·806	3·9	0·803	3·5
(b) CO_2										
218	436	0	0·661	0·633	−1·2	0·838	0·834	−0·5	0·835	−0·4
218	1308	0	0·608	0·606	−0·33	0·798	0·808	1·3	0·802	0·5
1110	555	0	0·697	0·683	−2·0	0·818	0·801	−2·1	0·813	−0·6
218	436	4	0·652	0·636	−2·5	0·759	0·825	8·7	0·813	7·1
218	1308	4	0·600	0·593	−1·2	0·775	0·800	3·2	0·794	2·5
555	555	4	0·648	0·635	−2·0	0·762	0·805	5·6	0·802	5·2
555	1110	4	0·619	0·607	−1·9	0·757	0·797	5·3	0·786	3·8
555	1665	4	0·595	0·592	−0·5	0·773	0·792	2·5	0·791	2·3
1110	555	4	0·669	0·658	−1·6	0·783	0·793	1·3	0·799	2·0
218	436	8	0·627	0·627	0	0·726	0·809	11·4	0·797	9·8
218	1308	8	0·585	0·575	−1·7	0·736	0·796	8·2	0·778	5·7
555	555	8	0·608	0·601	−1·2	0·757	0·793	4·8	0·789	4·2
555	1110	8	0·589	0·581	−1·4	0·752	0·790	5·1	0·782	4·0
555	1665	8	0·571	0·563	−1·4	0·754	0·789	4·6	0·778	3·2
218	436	12	0·595	0·588	−1·2	0·742	0·797	7·4	0·790	6·5
218	1308	12	0·564	0·560	−0·7	0·743	0·791	6·5	0·779	4·8

calculation of the heat-transfer coefficient the analogy expression relating heat transfer and skin friction is utilized:

$$\left(\frac{C_f}{2\,St}\right)^* = (Pr^*)^{\frac{2}{3}}. \qquad (4)$$

To compare the boundary-layer results with the reference temperature formula of equation (4) the value of $(C_f/2\,St)^*$ is determined from the exact calculations as follows:

$$\left(\frac{C_f}{2\,St}\right)^* = \left(\frac{C_f}{2\,St}\right)_s \frac{C_p^*}{C_{p_s}}. \qquad (5)$$

The values obtained from equations (5) and (4) are tabulated in columns (8) and (9) respectively of Table 2. The percentage error is less than 10 per cent in all cases except one, with the deviations in general being higher for CO_2 than for N_2. This agreement is as good as that reported in reference 1, and in the case of nitrogen it is somewhat better.

The alternative procedure proposed by Eckert is to define the heat-transfer coefficient in terms of enthalpy difference:

$$q_w = h_i(i_r - i_w) \qquad (6)$$

where now the enthalpy recovery factor r_i is given by

$$r_i = \frac{i_r - i_s}{(u_s^2)/2} \qquad (7)$$

The value of the enthalpy recovery factor r_i as taken from the boundary-layer solutions is reported in column 9 of Table 2 and compared with the approximate value given as $\sqrt{(Pr^*)}$ in column 10. The resulting error in the use of the reference method is comparable to that in reference [1], being of the order of 1 to 2 per cent.

The heat-transfer coefficient h_i is reported in dimensionless form as $(C_f/2\ St_i)$ and the boundary-layer calculations are shown in column 10 of Table 2. The reference method proposed a value of (Pr^*) for this ratio. The agreement of the approximate and exact values is even better than that reported in reference [1].

Conclusions and recommendation

The use of the reference temperature method yields approximations for N_2 and CO_2 laminar boundary layers which are as good as those of reference [1] for the air. If the specific heat varies over a wide range or if dissociation occurs it is recommended as in reference [1], though this remains to be verified, that properties be evaluated at a reference enthalpy rather than the reference temperature, where the reference enthalpy is given by:

$$i^* = i_s + 0{\cdot}5(i_w - i_s) + 0{\cdot}22(i_r - i_s).$$

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ON THE REPRESENTATION OF THE DENSITY DEPENDENCE OF THE THERMAL CONDUCTIVITY OF SUPERHEATED STEAM

A. DAS GUPTA and A. K. BARUA

Indian Association for the Cultivation of Science, Calcutta-32, India

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INTRODUCTION

Thermal conductivity data on superheated steam have a large number of practical applications. Consequently, the study of this property has recently drawn considerable attention. Kestin *et al.* [1] have attempted to represent the density dependence of the thermal conductivity of superheated steam by a second order polynomial in the densities. Such representation of viscosity and thermal conductivity has been tried by Curtis *et al.* [2] for moderately dense gases. However, their method cannot at present be applied to the data on steam as they did not consider association which plays a very significant role in steam [3].

Kestin *et al.* [1] considered the two most reliable sets of data reported by Keyes and Vines [4] (henceforth to be referred to as MIT data) and by Vargaftik *et al.* [5–9] (henceforth to be referred to as VTI data). However, it was